POWER BASICS®

Algebra

Robert Taggart

Table of Contents

LESSON 5: Solving Equations

GOAL: To learn how to simplify and solve equations and equations with squared variables

WORDS TO KNOW

constant	exponent	square root
cubed	formula	squared
denominator	numerator	squared variable
distance formula	reciprocal	

Solving Equations by Subtracting and Adding

When you solve an equation, you want to get the variable by itself. This way, you can find the value of the variable. Look at the equation below.

Example 1

$$x + 6 = 13$$

In this equation, you want to get the x by itself. To do this, you must get rid of the 6. Notice that 6 is being added to x. To get rid of the 6, do the opposite operation: Subtract 6. This is called doing the inverse operation. The word *inverse* means "opposite." If you subtract 6 from the left side, you must subtract 6 from the right side, too. Then both sides of the equation remain the same.

$$x + 6 - 6 = 13 - 6$$

$$x = 7$$

By subtracting the same number from both sides of the equation, you get x by itself: $x = 7$. The solution is 7.

Example 2

Try another equation: $m - 30 = 67$. In this equation, you want to get rid of the 30 so the m will be by itself. Notice that 30 is being subtracted from m. Do the inverse operation: Add 30. Do this to both sides of the equation.

$$m - 30 + 30 = 67 + 30$$

$$m = 97$$

TIP

When you solve for a variable, you can check to see if your answer is correct. Just put your answer in place of the variable in the original equation. Then see if the equation still works. Look at the example below.

$$x + 2 = 7$$

$$x + 2 - 2 = 7 - 2$$

$$x = 5$$

Now check your answer.

$$x + 2 = 7$$

$$5 + 2 = 7$$

$$7 = 7$$

Yes, your answer is correct!

PRACTICE 17: Solving Equations by Subtracting and Adding

Solve each equation below. Write your answer on the line after each equation.

1. $x + 3 = 24$ $x =$ _____
2. $z + 450 = 2467$ $z =$ _____
3. $h - .5 = 4$ $h =$ _____
4. $z - 36 = 683$ $z =$ _____
5. $q - 70 = 120$ $q =$ _____
6. $m - 12 = 45$ $m =$ _____

Solving Equations by Dividing and Multiplying

Equations can also be solved by using division and multiplication.

Example 1

Look at the following equation.

$$9x = 27$$

In the equation above, you must get rid of the 9 to get the x by itself.

Notice that 9 is being multiplied by x. To get rid of 9, do the inverse operation: Divide by 9. Do this to both sides of the equation.

$$\frac{9x}{9} = \frac{27}{9}$$

$$x = 3$$

You now know that you must get a variable by itself to find its value and solve an equation. You have learned that when a number is added to a variable in an equation, you must subtract that number from both sides of the equation. When a number is subtracted from a variable, you must add that number to both sides of the equation. When a variable is multiplied by a number in an equation, you must divide both sides by that number. Lastly, when a variable is divided by a number in an equation, you must multiply both sides by that number.

Look at the example below.

Example 2

$$\frac{x}{3} = 9$$

To get the x by itself, you must multiply both sides of the equation by 3.

$$\frac{3}{1}\left(\frac{x}{3}\right) = 3(9)$$

$$\frac{3x}{3} = 27$$

$$x = 27$$

PRACTICE 18: Solving Equations by Dividing and Multiplying

Solve each equation on the next page. Write your answer on the line after each equation.

Example: $6z = 24$

$$\frac{6z}{6} = \frac{24}{6}$$

$$z = 4$$

1. $10x = 150$ $x =$ _____

2. $84 = 3m$ $m =$ _____

3. $\frac{n}{4} = 12$ $n =$ _____

4. $\frac{f}{7} = 3$ $f =$ _____

5. $\frac{s}{18} = 5$ $s =$ _____

6. $\frac{y}{9} = 3$ $y =$ _____

IN REAL LIFE

Suppose it is your friend's birthday. You are in charge of collecting money for the birthday cake. You know that the cake costs $12.00. You also know that there are four people (including yourself) who want to help pay. How much should each person pay? If you think of this problem as an algebraic equation, it might look like this: $4x = 12$. x is the amount of money each person must pay. It is the unknown number. Now solve the equation. Divide both sides by 4 to get the x by itself. $\frac{4x}{4} = \frac{12}{4}$. $x = 3$. Now you know that each person should pay $3.00!

Solving Two-Step Equations

In some equations, the variable is part of two or more different operations.

Look at the example below.

Example 1

$$4x - 8 = 12$$

In this equation, 4 is multiplied by x. Then 8 is subtracted from the product of 4 and x. You still need to get the variable by itself. What do you do first?

Rule for Solving Two-Step Equations

If there are parentheses—( )—in an equation, always do the operation in parentheses first. When there is more than one operation and there are no parentheses, do the steps below:

1. Take care of any addition or subtraction operations.
2. Take care of any multiplication or division operations.

Example 2

Let's work through the following equation.

$$4x - 8 = 12$$

Step 1. First, take care of any addition or subtraction operations. Add 8 to both sides.

$$4x - 8 + 8 = 12 + 8$$

$$4x = 20$$

Step 2. Take care of any multiplication or division operations.

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

Now let's look at another example.

Example 3

$$\frac{x}{3} + 65 = 120$$

Step 1. Take care of any addition or subtraction operations. Subtract 65 from both sides of the equation.

$$\frac{x}{3} + 65 - 65 = 120 - 65$$

$$\frac{x}{3} = 55$$

Step 2. Take care of any multiplication or division operations. Multiply both sides by 3 to solve for x.

$$\frac{x}{3}(3) = 55(3)$$

$$x = 165$$

PRACTICE 19: Solving Two-Step Equations

Solve each equation below. Write your answer on the line.

Example:

$$2z - 5 = 11$$

$$2z - 5 + 5 = 11 + 5$$

$$2z = 16$$

$$\frac{2z}{2} = \frac{16}{2}$$

$$z = 8$$

1. $\frac{w}{7} + 12 = 33$ $\quad w =$ _____
2. $\frac{m}{6} - 8 = 14$ $\quad m =$ _____
3. $2p + 11 = 47$ $\quad p =$ _____
4. $7 + \frac{t}{6} = 15$ $\quad t =$ _____

POWER BASICS®

Algebra

Teacher's Guide

Table of Contents

Unit 3: Graphing Linear Equations

This unit introduces students to graphing linear equations on the coordinate plane. Lesson 7 presents graphing points and lines on the coordinate plane. Lesson 8 introduces linear equations and presents two approaches to graphing them on the coordinate plane: the table method and the intercept method. Lesson 9 introduces the students to linear slope, including the formula for finding slope, positive and negative slope, and how to graph a line using slope and one point.

Lesson 7—The Coordinate Plane

Goal: To graph ordered pairs on the coordinate plane

WORDS TO KNOW

axes	the horizontal number line (*x*-axis) and vertical number line (*y*-axis) on a coordinate plane; the singular is "axis."
axis	either of the number lines (horizontal and vertical) used to form a coordinate plane; the plural is "axes."
coordinates	a pair of numbers used to locate a point on a coordinate plane
coordinate plane	a flat surface divided into four parts by a horizontal line (*x*-axis) and a vertical line (*y*-axis) that meet in the center of the plane
horizontal	going from side to side
intersect	to cross at exactly one point
line	a straight path that goes on forever in two different directions
ordered pair	a pair of numbers used to locate a point on a coordinate plane, usually written inside parentheses; the first number tells how far to move horizontally and the second number tells how far to move vertically.
origin	the point at which the *x*-axis and the *y*-axis in the coordinate plane intersect
vertical	going up and down
***x*-axis**	the horizontal number line on a coordinate plane
***x*-coordinate**	the first number in an ordered pair that tells how far to move left or right from the origin
***y*-axis**	the vertical (up-and-down) number line on a coordinate plane
***y*-coordinate**	the second number in an ordered pair that tells how far to move up or down from the origin

Lesson 8—Graphing Linear Equations

Goal: To graph linear equations using the table method and the intercept method

WORDS TO KNOW

intercept method	graphing a linear equation by connecting the two points where the line meets the axes of the coordinate plane
linear equations	equations that have to do with lines

Lesson 9—Slope

Goal: To learn to find the slope of a line and to use a slope to graph lines

WORDS TO KNOW

ratio	the relationship between two numbers showing how they compare to each other
slope	the steepness of the slant of a line

Notes on Application Activity in Student Text

Activity	Skills Applied	Product
Coordinates and Maps	solving problems reading a map	list of coordinates optional activity: map and list of coordinates

Additional Activity Suggestions

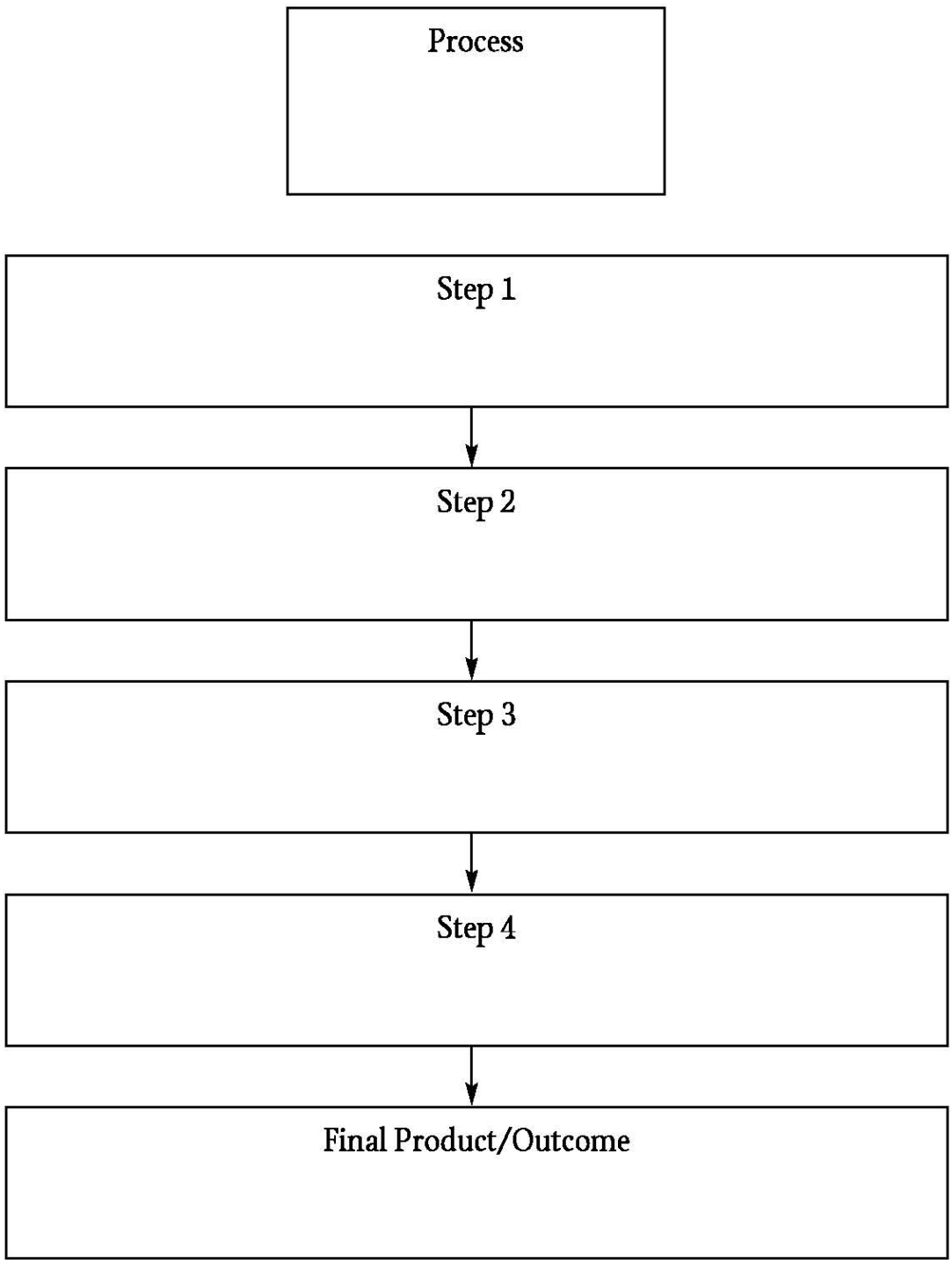
- Have learners build a number line in the form of a time line. Ask them to research a historical figure of their choice, or, if they prefer, interview a living person, and plot the major events of that person's life on a time line. The time line should have standard increments with events interspersed. You might have students include world events in order to give their subject's actions some context.

- With a large globe or an atlas handy, give learners the latitude and longitude of several locations and have them identify those locations. For example, ask learners "What city is located at latitude 38° 0' north and longitude 23° 44' east?" The answer is "Athens, Greece." In this activity learners could take turns posing questions to one another.

Differentiation

- Give students practice using coordinates to name points with this game, played in pairs. Each student marks one point in each quadrant of a coordinate grid, labeling the point in quadrant I A, in quadrant II B, in quadrant III C, and in quadrant IV D. Students should write the coordinates for each point below the grid. Holding their grids so that their opponents cannot see them, players take turns using coordinates to name one point in each quadrant. The first player to correctly guess the coordinates of all four of the other player's points wins.
- Ask students (working in small groups or as a whole class) to brainstorm a list of situations in which slope is found. If students have difficulty getting started, suggest roads, hills, ski resorts, and so forth. Have them say whether the slope is positive or negative in relation to sea level. Once the lists are completed, ask students to suggest ways that changing the slope would affect the situation. For example, how would reducing the slope of a ski run affect the difficulty of the run?

Steps in a Process Chart



POWER BASICS PLUS

Algebra

Workbook

Table of Contents

Unit 1: Algebra Basics

Unit 2: Solving Equations and Inequalities

Table of Contents, *continued*

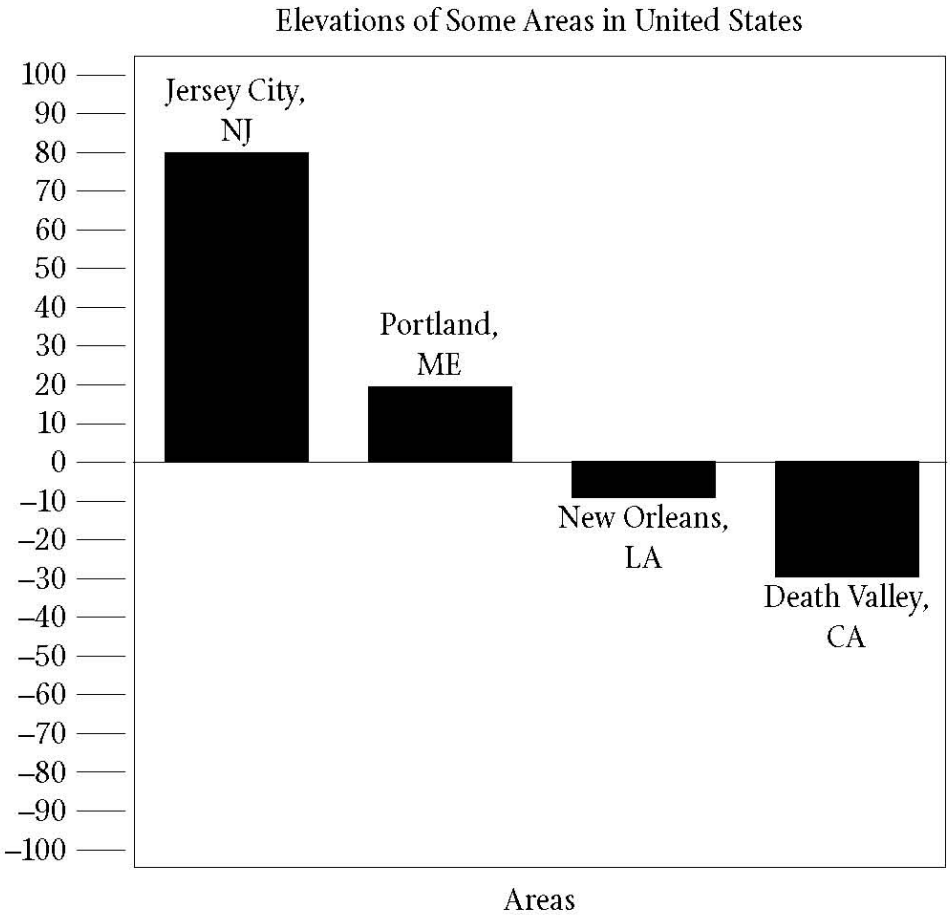
Table of Contents, *continued*

Unit 5: Quadratic Equations

NAME:

UNIT 1 • ACTIVITY 1
What Are Negative and Positive Numbers? I

Look at the graph below. Then answer the questions that follow.



1. What elevation in feet represents sea level? _____

2. What areas have elevations that are given in positive numbers? _____

_____

3. In relation to sea level, what do positive numbers represent? _____

4. What areas have elevations that are given in negative numbers? _____

_____

5. In relation to sea level, what do negative numbers represent? _____

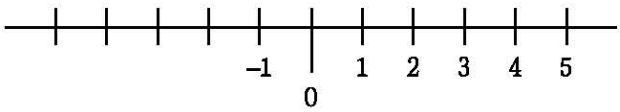
_____

NAME:

UNIT 1 • ACTIVITY 2
What Are Negative and Positive Numbers? II

Complete each number line by writing the missing negative numbers.

1.



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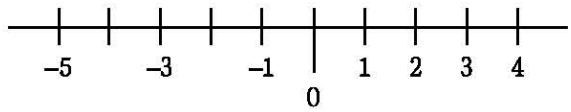

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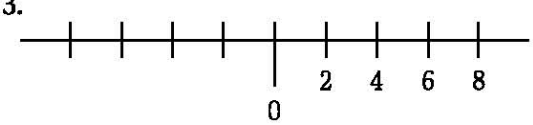

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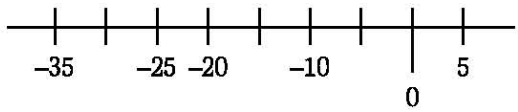
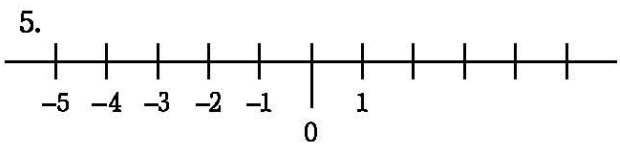


Complete each number line by writing the missing positive numbers.

5.



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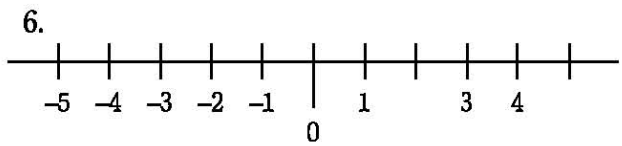

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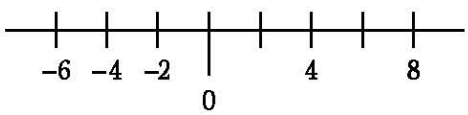

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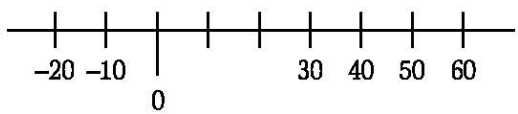
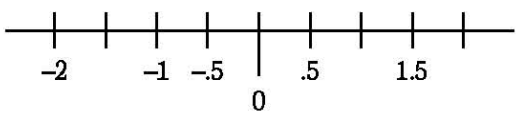


Complete each number line by writing the missing negative and positive numbers.

9.



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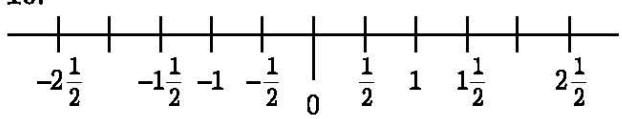


NAME:

UNIT 2 • ACTIVITY 35
The Distance/Rate/Time Formula I

Use the distance/rate formula, $D = r(t)$, to solve each problem below.

1. A bus travels 60 mph for 6 hours on the highway. How far did the bus travel?

   _____

2. A space shuttle travels at 500 mph for 10 hours. How many miles has the shuttle traveled?

   _____

3. Tami can rollerblade at a rate of 15 mph. If she rollerblades for 180 miles, how many hours has she rollerbladed? _____

4. A free-flying balloon travels at a rate of 20 mph. If it travels 460 miles before running into the side of a mountain, how many hours has the balloon been free flying? _____

5. Bianca flies in her glider for 4 hours and travels 256 miles. What is the average rate of speed of the glider? _____

6. A train traveled 1800 miles in 9 hours. How fast was the train going? _____

7. A sailboat travels 70 kilometers in 4 hours. What is its average rate of speed?

   _____

8. A truck driver drives at a speed of 65 mph for 6 hours on highway 95. How far has the driver traveled? _____

NAME:

UNIT 2 • ACTIVITY 36
The Distance/Rate/Time Formula II

Suppose you ride your bike to the library 8 miles away at rate of 4 mph on a calm day. However, when you ride your bike to the library on a windy day, you lose a mile an hour. Your rate is now 4 – 1 = 3 mph. You might think that the wind would have no effect on a round-trip time because the wind would speed up your bike ride from the library at a rate of 4 + 1 = 5 mph. However, the tables below show that the time it takes to make a round-trip does increase.

Using Distance ÷ Rate = Time

No wind

Rate		Time	Distance
One way	4 mph	2	8
Way back	4 mph	2	8

Total time = 2 + 2 = 4 hours

With wind

Rate		Time	Distance
One way	4 – 1 = 3 mph	$\frac{8}{3}$	8
Way back	4 + 1 = 5 mph	$\frac{8}{5}$	8

Total time = $\frac{8}{3} + \frac{8}{5} = 4\frac{4}{15}$ hours

Imagine you can paddle a canoe at 3 mph in still water. If there is a current, the canoe can travel at 4 mph going downstream but only 2 mph against the current on the way back. Fill in the tables to show how the time for a 6-mile round-trip changes from still water to water with a current.

1. Still water

	Rate	Time	Distance
One way			
Way back			

Total time = _____

2. Current

	Rate	Time	Distance
One way			
Way back			

Total time = _____

NAME:

UNIT 5 • ACTIVITY 100
Using the Quadratic Formula to Solve Word Problems

Use the quadratic formula to solve each word problem. Check your work by substituting your solutions into the variable of your original equation. Write your answer on the line.

1. The length of a rectangle is 6 more than its width. If the area is 16 square centimeters, what is the length and width of the rectangle?

   Length = _____

   Width = _____

   Check work:

2. The radius of a circle is 5 less than x. If the area of the circle is 36π square feet, what is the radius? (Use $A = \pi r^2$.)

   Radius = _____

   Check work:

POWER BASICS PLUS

Algebra

Test Pack

Table of Contents

Circle the correct answer for each of the following questions. Show your work, if necessary.

1. Solve the following equation.

$\frac{r}{12} + 7 = 73$

$r = ?$

a. $5\frac{1}{2}$

b. $6\frac{2}{3}$

c. 792

d. 960

---

2. Solve the following equation.

$12r + 17 = 113$

$r = ?$

a. 2

b. 8

c. 1152

d. 1560

---

3. Solve the following equation.

$\frac{2}{3}n + 2\frac{2}{3}n + 4 = 25 + \frac{1}{3}n$

$n = ?$

a. 5.7

b. 7

c. 63

d. 87

4. Solve the following equation.

$\frac{3}{4}w + 4\frac{3}{16}w + 9 = 24 - \frac{1}{16}w$

$w = ?$

a. 3
b. 6.6
c. 75
d. 165

---

5. $s = 5$ is the solution to which of the following equations?

a. $3\frac{3}{4}s + 3 = 5 - \frac{1}{4}s$

b. $\frac{2}{3}s - 3 = 5 - \frac{1}{3}s$

c. $7\frac{1}{2}s - 9 = 36 - \frac{1}{2}s$

d. $7\frac{1}{2}s + 9 = 36 + \frac{1}{2}s$

---

6. $s = 12$ is the solution to which of the following equations?

a. $3\frac{3}{4}s + 3 = 15 - \frac{1}{4}s$

b. $\frac{2}{3}s - 3 = 15 - \frac{1}{3}s$

c. $1\frac{1}{2}s - 9 = 45 + 1\frac{1}{2}s$

d. $2\frac{1}{2}s + 9 = 45 - \frac{1}{2}s$

---

7. Circle the letter of the inequality symbol that would make the following statement true.

$28 - 6$ _____ $2(6 + 4)$

a. >
b. ≤
c. =
d. <

8. Circle the letter of the inequality symbol that would make the following statement true.

$$\frac{13}{14} \text{ ——— } \frac{6}{7}$$

a. $<$

b. $>$

c. $=$

d. $\leq$

---

9. Beth is 7 years older than Darnell. Darnell is 2 times as old as Juan. Which formula represents Beth's age in relation to Juan?

a. $B = 2J - 7$

b. $B = 2J + 7$

c. $B = 2J + 14$

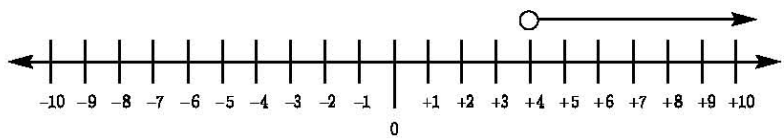
d. $B = J - 2$

---

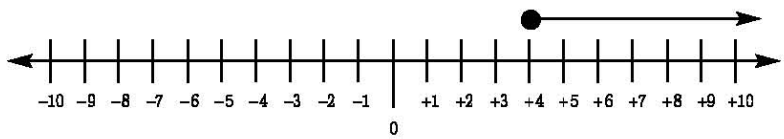
10. Which number line below shows the solution to the following inequality?

$$-\frac{1}{2}x + 12 > 28 + 3\frac{1}{2}x$$

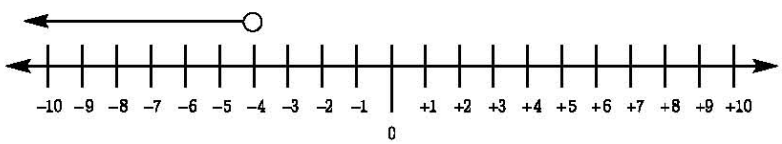
a.



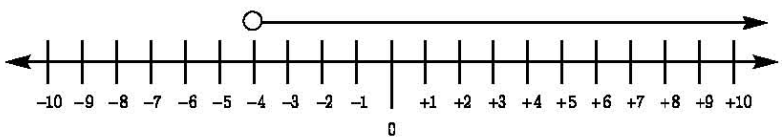
b.



c.



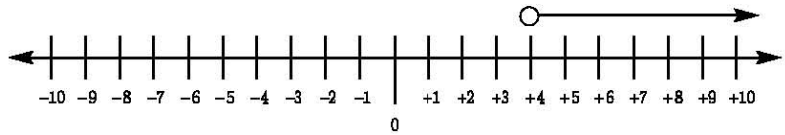
d.



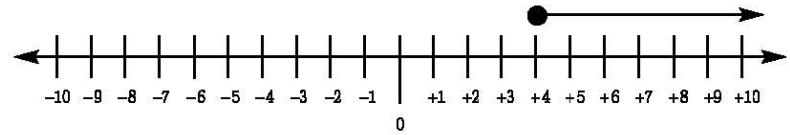
11. Which number line below shows the solution to the following inequality?

$x - 16 < 8 + 7x$

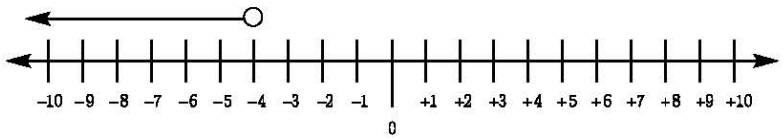
a.



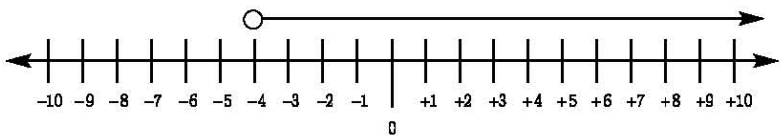
b.



c.



d.



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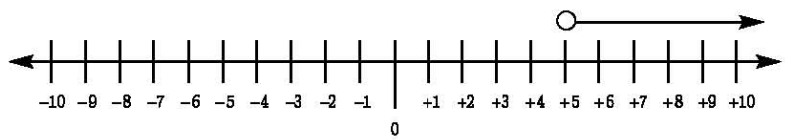
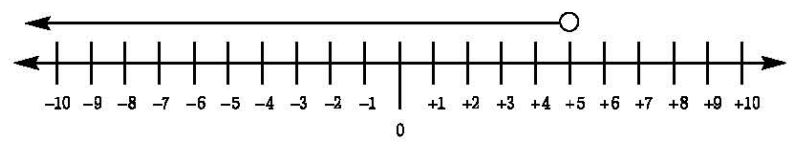
12. Which number line below shows the solution to the following inequality?

$-2y - 3 < 22 + 3y$

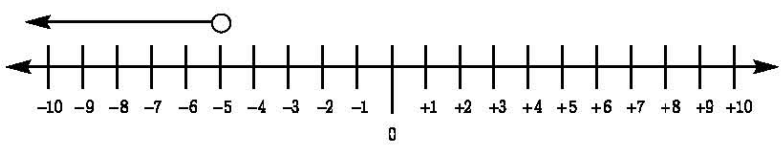
a.



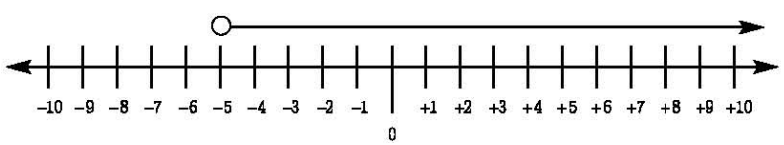
b.



c.



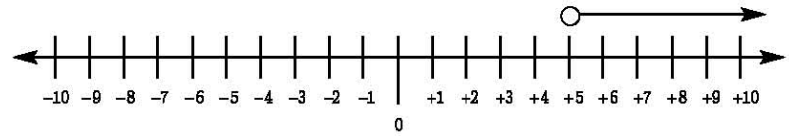
d.



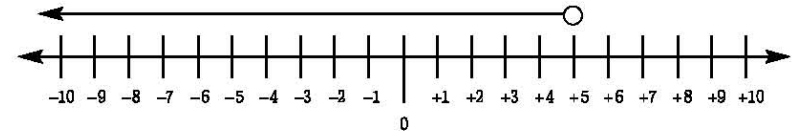
13. Which number line below shows the solution to the following inequality?

$-2y - 6 + y < 24 - 7y$

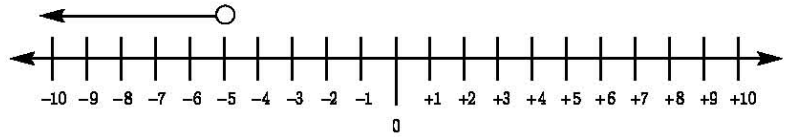
a.



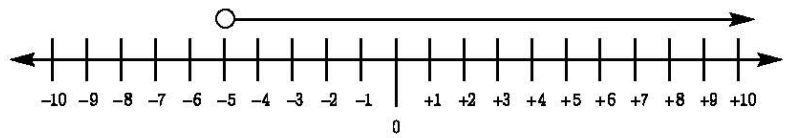
b.



c.



d.



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14. What is the solution to the following inequality?

$-3x + 5 > -1$

a. $x < 2$

b. $x > 2$

c. $x > 5\frac{1}{3}$

d. $x < 5\frac{1}{3}$

15. What is the solution to the following inequality?

$-3x + 5 < 11$

a. $x < -2$

b. $x > -2$

c. $x > 5\frac{1}{3}$

d. $x < 5\frac{1}{3}$

---

16. Which number below would NOT be a possible solution to the following inequality?

$8t - 12 > 16t + 4$

$t = ?$

a. -3

b. -4

c. -2

d. -12

---

17. Which number would NOT be a possible solution to the following inequality?

$7t - 12 > 16$

$t = ?$

a. 4

b. 12

c. 28

d. 36

18. Which number below would be a possible solution to the following inequality?

$-10m - 8 \geq -12m - 10$

$m = ?$

a. $-\frac{1}{2}$

b. $-1\frac{1}{2}$

c. -2

d. -12

---

19. Which number would NOT be a possible solution to the following inequality?

$4m - 8 \geq 12m - 10$

$m = ?$

a. -20

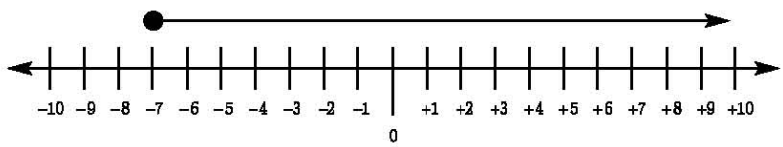
b. $-\frac{1}{2}$

c. 0

d. 2

---

20. Look at the number line below. Which of the following inequalities correctly describes the number line?



a. $d > 7$

b. $d \geq -7$

c. $d \geq 7$

d. $d < -7$