

### Geometry

**Robert Taggart** 



### **Table of Contents**

To the Student
Unit 1: Lines and Angles  Lesson 1: Points, Lines, and Dimensions Lesson 2: Angles Lesson 3: Equal Angles Lesson 4: Triangles Lesson 5: Right Triangles and the Pythagorean Theorem
Unit 2: Perimeter, Circumference, and Area  Lesson 6: Perimeter
Unit 3: Volume  Lesson 14: Volume of Rectangular Solids 13:  Lesson 15: Volume of a Pyramid 14:  Lesson 16: Volume of a Cylinder 14:  Lesson 17: Volume of a Sphere 15:  Lesson 18: Word Problems with Volume 15:
Unit 4: Coordinate Geometry  Lesson 19: Number Lines
Appendixes       220         A. Review of Triangles       220         B. Review of Circles       221         C. Review of Rules and Formulas       222         D. Abbreviations and Symbols       223         Glossary       225         Index       233

# UNIT 1

**Lines and Angles** 



#### **LESSON 1: Points, Lines, and Dimensions**

GOAL: To learn basic terms of geometry

#### **WORDS TO KNOW**

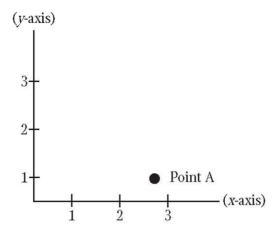
dimension	line segment	point
edges	parallel	ray
geometry	parallel lines	solid figure
line	plane	

#### **Basic Terms**

Geometry is a kind of math that deals with points, lines, angles, planes, and shapes. The word "geometry" literally means "the measurement of the world." Geometry is used to measure lines and shapes, and to show how they relate to one another. Geometry is used to build houses, bridges, and other structures, as well as in computer graphics, astronomy, and robotics. Everyday uses of geometry include figuring out how much carpet is needed to cover a floor, or how much water a fish tank will hold.

In this unit, you will learn about points, lines, angles, and planes.

Everything in geometry is a series of points. A **point** is a geometric element. A point has no length, width, or height. It can only be described by its position. A point is usually named by a capital letter, such as point A on the next page.



A **dimension** is a measure of length, width, or height. Points have no actual dimensions. In books, they are usually shown as a dot. The basic forms of geometry, such as lines and planes, are all built up of points.

A **line** is a set of points that are joined together. They have one dimension—length. They do not have width or height. Straight lines continue forever. In geometry books, a line is usually shown as a straight line with arrows on either end, like this:



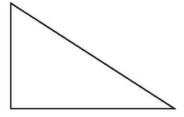
Lines are infinite. They continue forever in both directions. Usually, people do not work with the whole line. They work instead with a part of a line. A part of a line with a beginning and an end is called a **line segment**. Like a line, a line segment has only length. It does not have width or height. Line segments are usually shown with a dot at either end of the line segment, like this:



To show a line that continues forever on one end, but has an ending place on the other, you would use a **ray**. Rays, like lines

and line segments, have only one dimension: length. A ray is usually shown as a line with a dot at one end and an arrow at the other end, like this:

What if you joined three line segments to form a triangle shape? You would have a second dimension—width. As soon as you have width, you have a geometric element called a **plane**. A plane is a flat surface that has two dimensions, length and width. Unlike lines, plane figures are not infinite. They exist only in the area you can measure. This is what a plane figure looks like in a geometry book:



Plane figures can be in any flat shape you can think of—circles, squares, rectangles, triangles, and more. Any flat shape, whether it has curved lines or straight lines, is a plane figure.

A **solid figure** adds the third dimension of depth. Solid figures have length, width, and depth. Like a plane figure, a solid figure exists only in the area you can measure. You can measure its length, width, and depth. Because you are reading flat paper, you cannot really see a solid figure on a page. However, we can draw a figure to make it look more solid. Here is a drawing of a geometrical solid:

#### ■ PRACTICE 1: What Is Geometry?

Circle the correct word or phrase to complete each sentence below.

- 1. A point can only be defined by its
  - a. position.
- b. size.

c. shape.

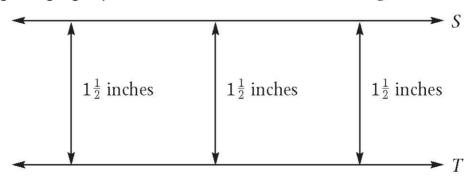
- **2.** A line continues
  - a. until it is stopped by a solid.
  - **b.** forever in both directions.
  - c. in all three dimensions.
- 3. A plane figure does not have
  - a. depth.
- b. length.
- c. width.
- **4.** A solid figure has \_\_\_\_\_ dimensions.
  - **a.** 2

**b.** 3

**c.** 5

#### **Parallel Lines**

Lines are one-dimensional. They have only one measurement—the measurement of length. In this unit, you will learn about a special property that some lines have. Look at the diagram below.



Look at the pair of lines above. The top line (line S) is  $1\frac{1}{2}$  inches from the bottom line (line T). If you made lines S and T in this diagram longer, the lines would still be  $1\frac{1}{2}$  inches apart.

The two lines would never cross, no matter how long you made them. Lines that never cross and stay the same distance apart are called **parallel lines**. Lines *S* and *T* on the preceding page are parallel lines.

#### THINK ABOUT IT

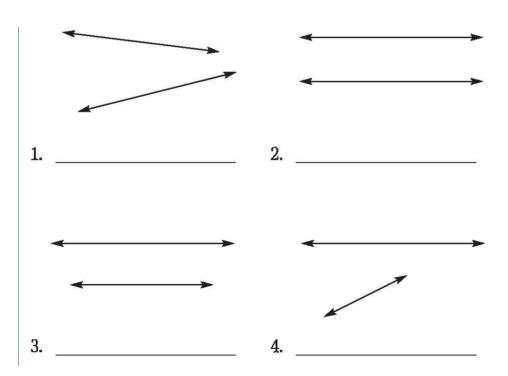


Look at the following line:

Is this line **parallel**? Why or why not? Can you tell from the information given? Write your answer on a separate sheet of paper.

#### ■ PRACTICE 2: Parallel Lines

Look at each pair of lines below. Decide if the lines are parallel. If the lines are parallel, write *parallel* on the line below the diagram. If the lines are not parallel, write *not parallel* on the line below the diagram.





# Geometry

Teacher's Guide



## **Table of Contents**

To the Teacher
Classroom Management
Unit 1: Lines and Angles
Unit Overview
Additional Activity Suggestions
Unit 2: Perimeter, Circumference, and Area
Unit Overview5
Additional Activity Suggestions
Unit 3: Volume
Unit Overview
Additional Activity Suggestions
Unit 4: Coordinate Geometry
Unit Overview
Additional Activity Suggestions
Answer Key
Graphic Organizers
Student Book Appendixes30
Student Book Glossary

### **Unit 1: Lines and Angles**

This unit introduces the study of geometry. In Lesson 1, students learn the basic terms of geometry, such as dimensions, points, and lines. In Lesson 2, they begin to learn about angles, including right angles, complementary angles, and supplementary angles. Lesson 3 continues the exploration of angles, introducing students to naming angles, equal angles, and finding the measurements of angles. Lesson 4 moves on to the study of triangles, with a definition of a triangle and an explanation of the ways to describe triangles. Lesson 5 introduces students to the Pythagorean theorem.

#### **Lesson 1—Points, Lines, and Dimensions**

Goal: To learn basic terms of geometry

#### WORDS TO KNOW

**dimension** a measure in one direction, such as length, width, or height

edges the line segments where two faces of a solid figure meet

**geometry** the area of mathematics that deals with the measurement and relationship of

points, lines, angles, solids, and surfaces

**line** a straight path that goes on forever in two different directions

**line segment** a part of a line that includes two points, called endpoints, and all the points

between the endpoints

parallel lying in the same plane but not touching at any point

parallel lines lines that are always the same distance apart but never meet

plane a flat surface or area

**point** an exact location in space, usually represented by a dot

ray part of a line; it has one endpoint and continues without end in one direction

solid figure a three-dimensional shape

#### Lesson 2—Angles

Goal: To learn properties of different types of angles

#### **WORDS TO KNOW**

angles figures formed by two lines that extend from the same point

**complement** the complement of an angle is the angle that, when added to the first

angle, totals 90°

**complementary angles** two angles whose measures add up to 90°

degrees units for measuring angles, shown with the symbol °; based on

dividing a circle into 360 equal parts

perpendicular meeting at a right angle

right angles angles whose measure is 90°

straight angle an angle that measures 180°

**supplement** the supplement of an angle is the angle that, when added to the first

angle, totals 180°

supplementary angles two angles whose measures add up to 180°

#### **Lesson 3—Equal Angles**

Goal: To find equal angles and figure out the measurements of angles based on their relationships to other angles

#### WORD TO KNOW

transversal a line that crosses two or more lines at different points

#### Lesson 4—Triangles

Goal: To identify different types of triangles and find the measurements of angles in a triangle

#### WORDS TO KNOW

acute angle an angle that has a measure greater than  $0^{\circ}$  and less than  $90^{\circ}$ 

acute triangle a triangle in which all three angles are acute, that is, greater than 0°

and less than 90°

equilateral triangle a triangle where all three sides are the same length isosceles triangle a triangle in which two sides are the same length

obtuse angle an angle that has a measure greater than  $90^{\circ}$  and less than  $180^{\circ}$ 

**obtuse triangle** a triangle that has one obtuse angle (one angle that measures greater

than 90° and less than 180°)

plane figure a figure that lies on one plane; it has only two dimensions

right triangle a triangle that has one right angle (an angle that measures 90°)

scalene triangle a triangle in which no two sides are the same length

**triangle** a flat shape with three sides

**two-dimensional** measured in two dimensions, or directions, such as length and width; flat

#### Lesson 5—Right Triangles and the Pythagorean Theorem

Goal: To use the Pythagorean theorem to find the lengths of the sides of right triangles

#### **WORDS TO KNOW**

formula a general rule for finding the value of something; often written with variables

**hypotenuse** the side of a right triangle that is opposite the right angle

legs in a right triangle, the two sides that form the right (90°) angle

Pythagorean theorem a statement that says that, in any right triangle, the square of the side opposite

the right angle (the hypotenuse) is equal to the sum of the squares of the other two sides. If one side is 2 cm long and the other side is 3 cm long, then the

square of the hypotenuse is  $2^2 + 3^2 = 4 + 9 = 13$ .

square a number multiplied by itself

**square root** The square root of a number is the factor that, when multiplied by itself, gives

the number.

**square root symbol** The symbol for "square root of" is  $\sqrt{\phantom{a}}$ , as in  $\sqrt{9} = 3$ .

theorem an important mathematical statement that can be proved to be true

#### Notes on Application Activities in Student Text

Activity	Skills Applied	Product
Finding Lines and Angles	gathering information	drawings
	preparing visual demonstrations	
Triangle Angles	visualizing shapes	reconfigured triangle
	working with others	written paragraph

#### **Additional Activity Suggestions**

■ People who work in the building trades work with lines and angles a great deal. Have learners contact a builder or carpenter, and ask what specific skills (such as measuring and calculating) and tools (such as levels and T-squares) are used to make sure a project is done accurately and holds together. Learners could also have a builder or carpenter demonstrate how to use these tools, or learners could demonstrate this themselves.



#### Teaching Tip

■ To reinforce identification of various types of triangles, have learners search their school, home, workplace, and so on for examples of scalene, equilateral, and isosceles triangles. Have them bring in pictures or drawings of five examples of each. They should also note which are also right triangles.



#### Differentiation

- Students learning geometry can get caught up in a slew of definitions, propositions, theorems, formulas, and so on. All the numbers and symbols can make everything seem very abstract. You can help learners see how geometry is connected to reality by taking them on a mini-field trip through the building. Have them observe structural congruencies, examples of parallelism, the way components of the building are made up of the figures they are studying, and so on. This should help them realize that geometry is real. It is everywhere. It is not just a bunch of formulas and theorems. Once students can recognize and name geometrical figures, they'll feel less intimidated to work with them.
- Preview the vocabulary in each lesson by reading the Words to Know and their definitions to your students. For each definition, point to an object in the classroom that fits the definition. Then ask students to identify other objects that also fit the definition. This helps them have a concrete understanding of the new concepts.

### **Graphic Organizers**

#### **Graphic Organizers**

Graphic organizers are a versatile teaching and learning tool. They can help students clarify their thinking, integrate new knowledge, reinforce their understanding of a topic, and review material for quizzes and tests. Using graphic organizers, learners can understand content more clearly and can take clear, concise notes. Graphic organizers can also act as a visual aid to make abstract concepts more concrete.

The graphic organizers provided here can be used in many ways. You can use transparencies of the organizers to introduce or review a topic with the entire class. You can photocopy the organizers and allow students to use them as they work through the student text. Here is a brief description of the organizers in this section and their uses.

#### Structured Notes

This organizer is one way of organizing notes as students read through the text. Students should write the main topic in the box at the top. In the boxes underneath they can write details about the topic, specific information, examples, and so forth.

#### Concept and Definition Chart

This chart is used to keep track of new vocabulary and concepts as they are introduced in the text. Students should write the word or concept in the box at the top of the chart. They should then fill in the information in the rest of the boxes.

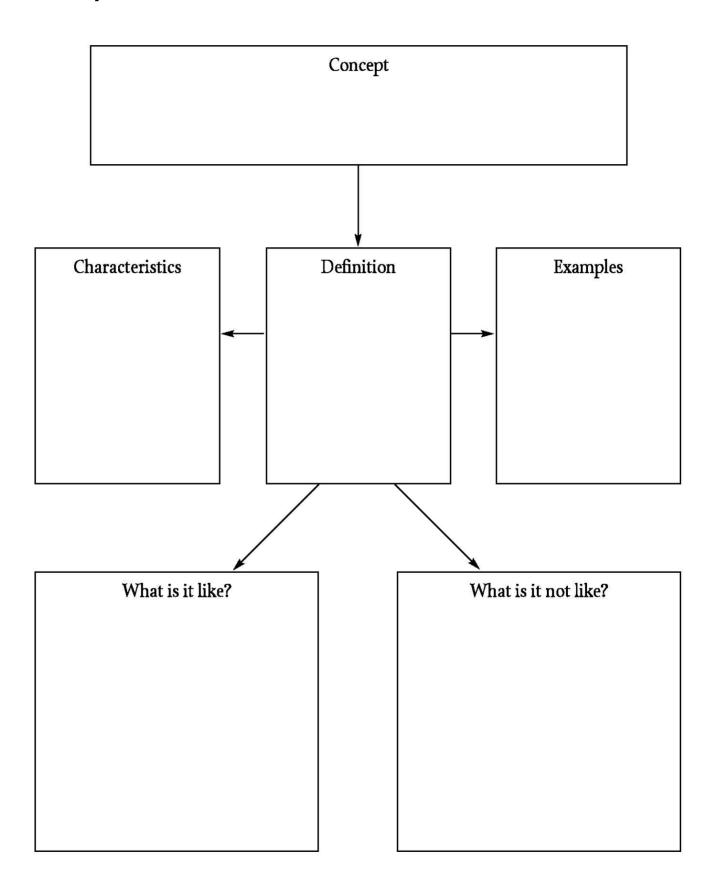
#### Steps in a Process Chart

This graphic organizer is used to show information in order. Students will find this organizer particularly useful when taking notes of mathematical processes, showing the steps in order. They should write the process in the box at the top of the chart, then break the process down into steps and write one step in one box, adding or deleting boxes as needed.

#### Table

This graphic organizer has many uses. Students should label each column, then write relevant information in each cell of the chart.

### **Concept and Definition Chart**





# Geometry

Workbook



### **Table of Contents**

To the Studentvii	Activity 18  Drawing Transversals
Unit 1: Lines and Triangles	Activity 19
Activity 1	Regular and Irregular Polygons19
Dimensions1	Activity 20
Activity 2	More Polygons
Parallel Lines2	Activity 21
Activity 3	Quadrilaterals
Measuring Line Segments	Activity 22  Acute and Obtuse Triangles22
Activity 4 Constructing Congruent Line Segments	· ·
Using a Compass	Activity 23  The Sum of the Angles of a Triangle23
Activity 5	Activity 24
Constructing Congruent Line Segments	The Sum of the Angles in a Quadrilateral 24
Using a Safe-Drawing Compass5	Activity 25
Activity 6	Exponents
Angles, Angles Everywhere! 6	Activity 26
Activity 7 Acute, Obtuse, and Straight Angles	Squares and Square Roots26
Activity 8	Activity 27
Measuring Angles8	Drawing the Pythagorean Theorem27
Activity 9	Activity 28
Constructing Perpendicular Lines 9	Pythagoras
Activity 10	Unit 2: Perimeter, Circumference, and Area
Using Equations to Find Complements 10	Activity 20
Activity 11	Activity 29 Using Formulas to Find the Perimeter
Using Equations to Find Supplements 11	of Rectangles and Squares29
Activity 12	Activity 30
Complementary and Supplementary Angles . 12	Perimeter of Regular Polygons 30
Activity 13  Drawing Angles	Activity 31
Activity 14	Area and Blueprints31
Angle Diagram	Activity 32
Activity 15	Relating Area and Perimeter
Vertical and Adjacent Angles	Activity 33  Finding the Area of a Parallelogram I33
Activity 16	Activity 34
Constructing Parallel Lines 16	Finding the Area of a Parallelogram II 34
Activity 17	0
Street Angles 17	



# Table of Contents, continued

Activity 35 Finding the Area of a Triangle	Activity 53
	Algebra, Formulas, and Circles53
Activity 36 Finding the Area of a Trapezoid	Activity 54 Using the Area and Circumference Formulas . 54
Activity 37	Activity 55
Finding the Area of a Complex Figure 37	Word Problems with Perimeter,
Activity 38	Circumference, and Area55
Drawing Circles	Unit 3: Volume
Activity 39	Ont o. Volume
Radii, Circumference, and Arcs I 39	Activity 56
Activity 40	Space Figures56
Radii, Circumference, and Arcs II40	Activity 57
Activity 41	Polyhedra57
Central Angles	Activity 58
Activity 42	Platonic Solids58
Using Equations to Find Central Angles42	Activity 59
Activity 43	Finding Volume
Central Angles and Fractions43	Activity 60
Activity 44	Finding the Volume of Rectangular
More about $Pi(\pi)$	Prisms and Cubes
Activity 45	Activity 61
Perimeter of Complex Figures45	Finding the Volume of Prisms
Activity 46	Activity 62
Percents	Visualizing the Volume of a Cylinder 62
Activity 47	Activity 63
Making Circle Graphs 47	Nets63
Activity 48	Activity 64
Area of a Circle48	Finding the Volume of Cones
Activity 49	and Pyramids64
Using Radius or Diameter to Find	Activity 65
the Area of a Circle49	Finding the Volume of Space Figures 65
Activity 50	Activity 66
Area of Complex Figures 50	Surface Area of Rectangular Prisms 66
Activity 51	Activity 67
Changing Formulas51	Surface Area of Cylinders67
Activity 52	Activity 68
Using Formulas52	Word Problems with Volume 68



# Table of Contents, continued

Onit 4. Coordinate Geometry
Activity 69 Fractions on the Number Line
Activity 70 Property of Density70
Activity 71  Graphing the Solution to an Equation or an Inequality on the Number Line71
Activity 72 Drawing the Coordinate Plane72
Activity 73 The Coordinate Plane
Activity 74 Graphing on the Four Quadrants of the Coordinate Plane
Activity 75 Graphing Game
<b>Activity 76</b> Transformations on the Coordinate Plane 76
Activity 77 Functions
Activity 78 Linear Equations
Activity 79 Using Intercepts to Graph Linear Equations
Activity 80 Graphing Nonlinear Equations80
Activity 81 Functions and Measurement
Activity 82 Graphing Inequalities82
<b>Activity 83</b> Slope I
Activity 84 Slone II 84

Activity 85
The Slope-Intercept Form of an
Equation of a Line
Activity 86
Graphing a Line Using the Slope-
Intercept Form of an Equation 86
Activity 87
Slope and Parallel and
Perpendicular Lines
Activity 88
Systems of Equations88
Activity 89
Graphing Equations and Inequalities
to Solve Word Problems



#### **UNIT 1 • ACTIVITY 1**

#### Dimensions

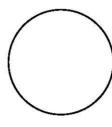
A dimension is a measure of length, width, or height. Plane figures, such as rectangles and squares, have width and length. They are two-dimensional. Objects, such as your classroom, a car, or a table, have length, width, and height. They are three-dimensional.

Look at each picture below. Decide if the object that is pictured has two dimensions or three dimensions. Write *two-dimensional* or *three-dimensional* on the line below each picture.

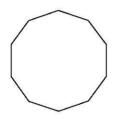
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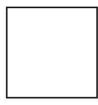
2.



3.



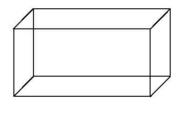
4.



5.

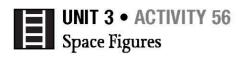


6.



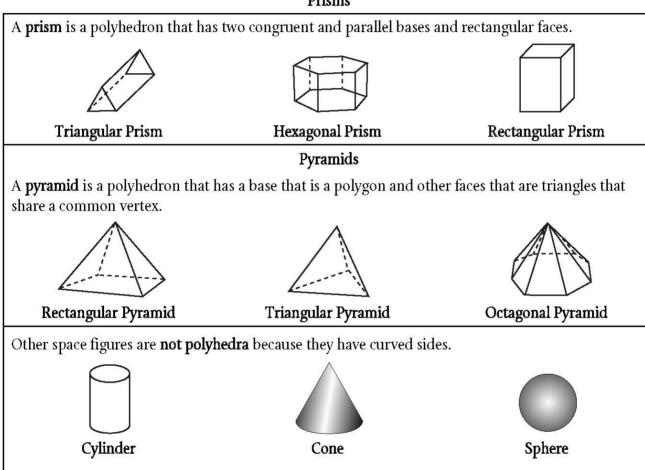
List three other things that are three-dimensional. Then list three other things that are two-dimensional.

7. \_\_\_\_\_\_



A space figure is a figure with three dimensions: length, width, and height. Space figures are also called solid figures and three-dimensional figures. A polyhedron (plural *polyhedra*) is a space figure that is made up of polygons. You have learned about the polyhedron that is made up of rectangles. This figure is called a rectangular prism. Polyhedra are named for the shape of their bases. There are many space figures. Look at the chart below.

#### Prisms



Write the name of each figure on the line.

 1.
 2.
 3.
 4.

 5.
 6.
 7.
 8.



# Geometry

**Test Pack** 



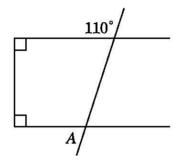
## **Table of Contents**

To the Teacher
Testing Students Who Do Not Test Well
Test-Taking Strategies for <i>Power Basics</i>
Pretest1
Unit 1 Test: Lines and Angles 8
Unit 2 Test: Perimeter, Circumference, and Area
Unit 3 Test: Volume
Unit 4 Test: Coordinate Geometry
Posttest
Answer Key
Student Record-Keeping Form
Strategies for Standardized Testing

#### **GEOMETRY • PRETEST**

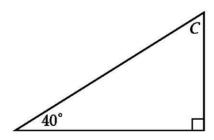
Circle the correct answer for each of the following questions. Show your work, if necessary.

**1.** How many degrees are there in angle *A*?



- **a.** 70°
- **b.** 110°
- **c.** 290°
- **d.** 90°

**2.** How many degrees are there in angle *C*?

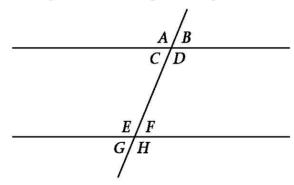


- **a.** 50°
- **b.** 40°
- **c.** 30°
- **d.** 25°

**3.** A right triangle has one leg that is 10 inches long and one leg that is 5 inches long. How long is the triangle's hypotenuse?

- **a.** 11.18 in.
- **b.** 1.25 in.
- **c.** 10 in.
- **d.** 15 in.

- 4. A triangle has two sides of equal length. With this information only, we can say that it is
  - a. a scalene triangle.
  - b. an equilateral triangle.
  - c. an isosceles triangle.
  - d. a right triangle.
- **5.** A triangle has one angle that is greater than 90°. With this information only, we can say that it is
  - a. a scalene triangle.
  - b. an acute triangle.
  - c. a right triangle.
  - d. an obtuse triangle.
- 6. A triangle has three sides of equal length. With this information only, we can say that it is
  - a. an equilateral triangle.
  - b. an acute triangle.
  - c. both an equilateral and an acute triangle.
  - d. a right triangle.
- 7. Which angles below are equal to angle H?



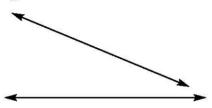
- **a.** ∠F, ∠E, ∠G
- **b. Z**G, **Z**D, **Z**C
- c. **Z**E, **Z**D, **Z**A
- **d.** ∠E, ∠C, ∠B
- **8.** Jonah wants to buy carpeting for his new apartment. He wants to carpet his living room and his bedroom. The living room is 6 meters long and 4 meters wide. The bedroom is 5 meters square. How much carpeting does Jonah have to buy to cover the floors in both rooms?
  - **a.**  $75 \text{ m}^2$
  - **b.** 49 m<sup>2</sup>
  - **c.** 36 m<sup>2</sup>
  - **d.** 15 m<sup>2</sup>

#### **UNIT 1 TEST • LINES AND ANGLES**

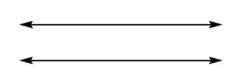
Circle the correct answer to each of the following questions. Show your work, if necessary.

- 1. Which of the following objects is two-dimensional?
  - a. a pair of dice
  - b. a basketball
  - c. a tree
  - d. a circle
- 2. Which pair of lines below are parallel?

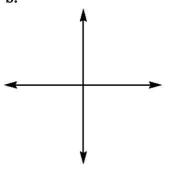
a.



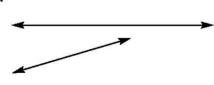
c.



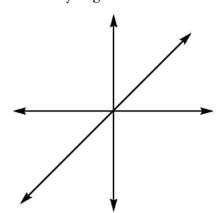
b.



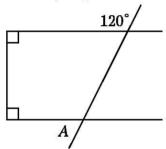
d.



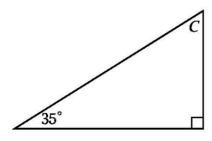
3. How many angles are there in the diagram below?



- **a.** 3
- b. 1
- **c.** 6
- d. 4
- **4.** How many degrees are there in angle *A*?



- **a.** 60°
- **b.** 120°
- **c.** 240°
- **d.** 90°
- **5.** How many degrees are there in angle C?

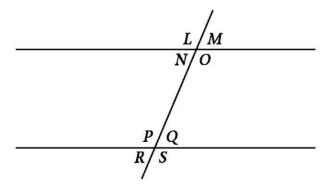


- **a.** 55°
- **b.** 90°
- **c.** 145°
- **d.** 180°

#### **GEOMETRY • POSTTEST**

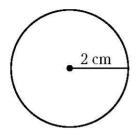
Circle the correct answer to each of the following questions. Show your work, if necessary.

- **1.** Angle *A* of triangle *ABC* measures 55° and angle *B* measures 40°. How much does angle *C* measure?
  - **a.** 65°
  - **b.** 75°
  - c. 85°
  - **d.** 95°
- 2. What is a triangle with no equal sides called?
  - a. an equilateral triangle
  - b. an obtuse triangle
  - c. a scalene triangle
  - d. an isosceles triangle
- 3. What is the complement of an angle that measures 45°?
  - **a.** 30°
  - **b.** 45°
  - **c.** 150°
  - **d.** 330°
- **4.** Which angles below are equal to angle *L*?



- a.  $\angle M, \angle P, \angle Q$
- b.  $\angle O$ ,  $\angle Q$ ,  $\angle R$
- c.  $\angle N$ ,  $\angle P$ ,  $\angle R$
- d.  $\angle O$ ,  $\angle P$ ,  $\angle S$

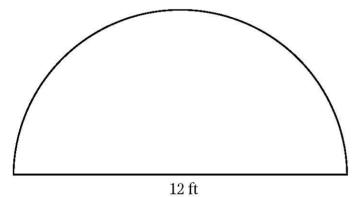
5. What is the area of the circle below? ( $\pi = 3.14$ )



- **a.**  $12.56 \text{ cm}^2$
- **b.** 12.56 cm
- **c.** 6.28 cm
- **d.** 6.28 cm<sup>2</sup>
- **6.** What is the circumference of a circle with a radius of 5 inches? ( $\pi = 3.14$ )

DATE:

- **a.** 15.7 meters
- **b.** 23.6 meters
- **c.** 31.4 meters
- **d.** 35.4 meters
- 7. What is the radius of a circle with a circumference of 62.8 meters? ( $\pi = 3.14$ )
  - **a.** 2.5 meters
  - **b.** 5 meters
  - c. 10 meters
  - d. 25 meters
- **8.** The base of the semicircle below is 12 feet. What is its area?  $(\pi = 3.14)$



- a. 30.84 square feet
- b. 37.68 square feet
- c. 56.52 square feet
- d. 78.84 square feet