

Daily Warm-Ups

ALGEBRA

NCTM Standards

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Introduction



I taught mathematics for many years at the middle- and high-school level. I know how precious every moment can be in a class period. The problems, exercises, activities, reflections, and writing prompts in this collection of warm-ups are just what I wish I had during those years. These warm-ups are organized in five parts around four strands of algebra designated by the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*: using mathematical models; analyzing change; using algebraic symbols; and understanding patterns, relations, and functions. The NCTM process standards, especially communication and representation, are also integrated within these activities.

The warm-ups address all algebra standards within NCTM, and exceed Algebra I, include Algebra II, and touch upon Trigonometry. The NCTM expectations within each topic area for grades 6 through 8 and grades 9 through 12 are addressed at least twice in different ways. Some problems address more than one standard or expectation, but have been placed in the respective section according to the topic area that I thought was best addressed by the activity.

The warm-ups are organized by standards rather than by level of difficulty. Use your judgement to select appropriate problems for your students. The problems are not meant to be done in consecutive order from the beginning of the book to the end. Some of these problems are stand-alone, some can launch a topic, some can be used for journal prompts, and some refresh students' skills and concepts. All are meant to enhance and complement high-school algebra curriculum programs. They do so by providing resources for teachers for those short spaces of time of 5 to 15 minutes when class time might go unused.

—Betsy Berry, Ph.D.

Part 1: Developing Algebraic Thinking Through and for Mathematical Modeling

National Council of Teachers of Mathematics: “Instructional programs from pre-kindergarten through grade 12 should enable all students to use mathematical models to represent and understand quantitative relationships.”

Expectations

Grades 6–8:

- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Grades 9–12:

- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.
- Draw reasonable conclusions about a situation being modeled.

Direct or Inverse?

Look at the data sets below. Decide if each one represents a situation that would give a direct proportion as a mathematical model, an inverse proportional representation, or neither. Explain your thinking for each one.

1.

x	3	-10	-2	2.5	20
y	1.67	-0.5	-2.5	0.2	0.25

2.

x	6	12	4	-3	24
y	2	1	3	-4	0.5

3.

x	-21	-3	6	12	15
y	-14	-2	4	8	10

4.

x	8	-12	-2	-3	4
y	$-\frac{1}{2}$	$-\frac{4}{3}$	-1	$-\frac{7}{3}$	0



Part 2: Analyzing Change in Various Contexts

National Council of Teachers of Mathematics: “Instructional programs from pre-kindergarten through grade 12 should enable all students to analyze change in various contexts.”

Expectations

Grades 6–8:

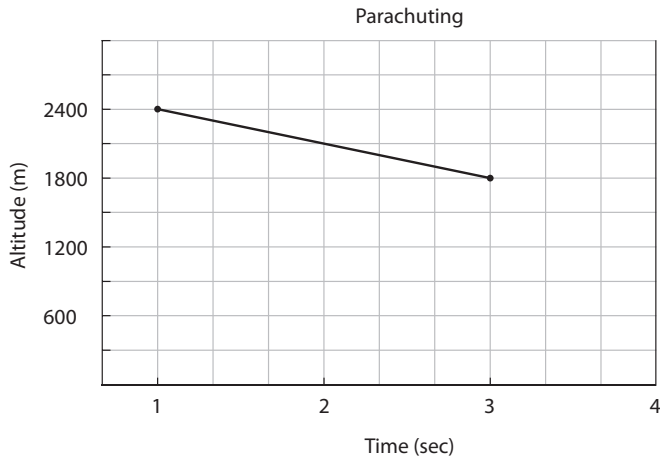
- Use graphs to analyze the nature of changes in quantities in linear relationships.

Grades 9–12:

- Approximate and interpret rates of change from graphical and numerical data.

Parachuting Down

Cecelia took her first parachute jump lesson last weekend. Her instructor gave her the graph below that shows her change in altitude in meters during a 2-second interval. Use the graph to answer the questions that follow.



1. What is the slope of the line segment?
2. Estimate Cecelia's average rate of change in altitude in meters per second.
3. Give the domain and range for this graph.



Part 3: Using Algebraic Symbols I

National Council of Teachers of Mathematics: “Instructional programs from pre-kindergarten through grade 12 should enable all students to represent and analyze mathematical situations and structures using algebraic symbols.”

Expectations

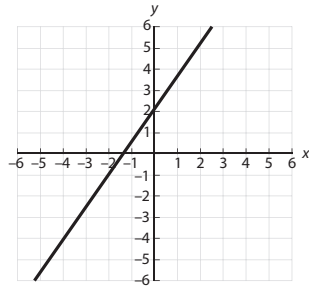
Grades 6–8:

- Develop an initial conceptual understanding of different uses of variables.
- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
- Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.

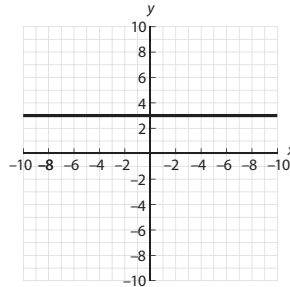
Up and Down the Line I

Write an equation for the line pictured in each graph below.

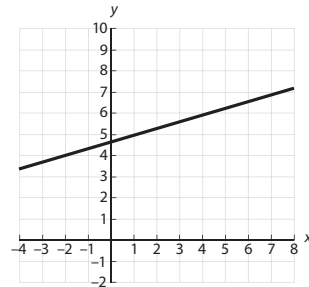
1.



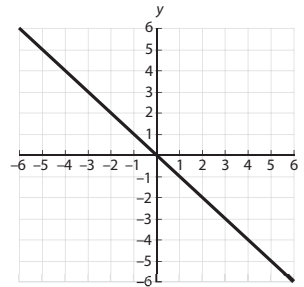
2.



3.



4.



Part 4: Using Algebraic Symbols II

National Council of Teachers of Mathematics: “Instructional programs from pre-kindergarten through grade 12 should enable all students to represent and analyze mathematical situations and structures using algebraic symbols.”

Expectations

Grades 9–12:

- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
- Use symbolic algebra to represent and explain mathematical relationships.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Bicycle Sales Matrices

The first matrix below represents the combined quarterly sales of three brands of mountain bikes and hybrid bikes at Bikes Unlimited in three large American cities. The second matrix shows the quarterly sales of hybrid bikes.

Combined Sales

	Trek	K2	Schwinn
Los Angeles	150	95	105
Boston	75	75	90
Chicago	85	110	175

Hybrid Sales

	Trek	K2	Schwinn
Los Angeles	50	40	70
Boston	30	35	50
Chicago	55	50	150

Find the matrix that represents the quarterly sales of mountain bikes. Find the matrix that represents the quarterly sales of mountain bikes and hybrid bikes in Boston for the three brands. Then construct one matrix that shows the total sales for all three brands in all three cities combined. Label the rows of your matrix with the brand names. Label the columns with the terms *hybrid* or *mountain*. What matrix operations did you use to construct each of these matrices?



Part 5: Understanding Patterns, Relations, and Functions

National Council of Teachers of Mathematics: “Instructional programs from pre-kindergarten through grade 12 should enable all students to understand patterns, relations and functions.”

Expectations

Grades 6–8:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship.
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Grades 9–12:

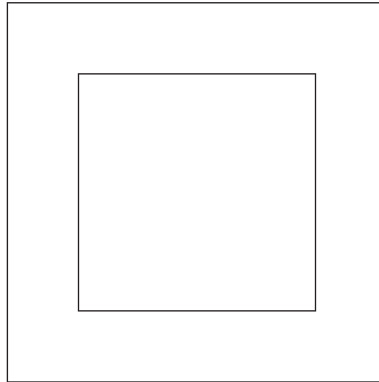
- Generalize patterns using explicitly defined and recursively defined functions.
- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions.

Part 5: Understanding Patterns, Relations, and Functions, *continued*

- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
- Interpret representations of functions of two variables.

Pool Border

Landscapers often use square tiles as borders for garden plots and pools. The drawing represents a square pool for goldfish surrounded by 1-foot square tiles.



1. How many tiles will be needed for the border of this pool with an edge of length S feet?
2. Express the total tiles in as many different ways as you can. Be ready to explain why your different ways are equivalent.

